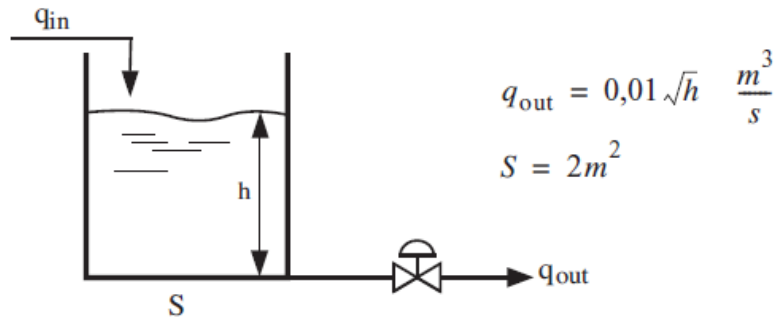


Series 3

Exercise 1

Consider the following reservoir:

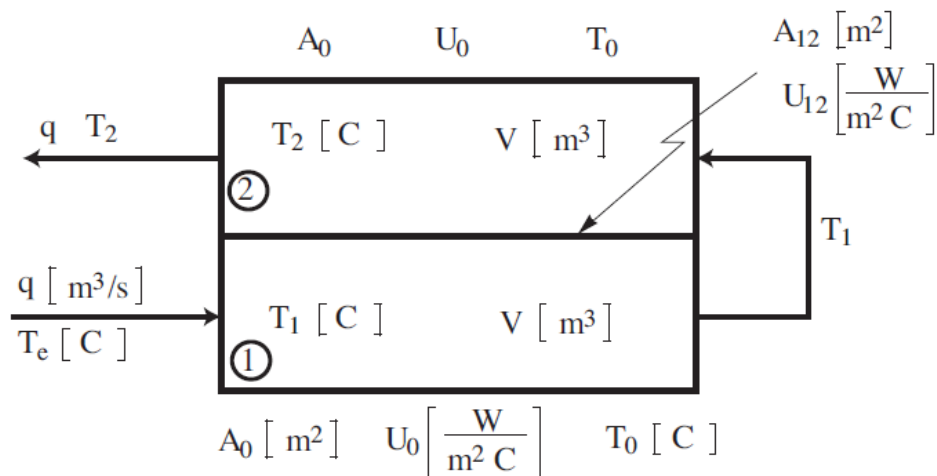


At time 0, while the system is in a stationary state $q_{in} = 0.015 m^3/s$, the feed flow is cut.

- Calculate the time necessary to empty half of the reservoir.
- Calculate the time necessary to empty the whole reservoir.

Exercise 2

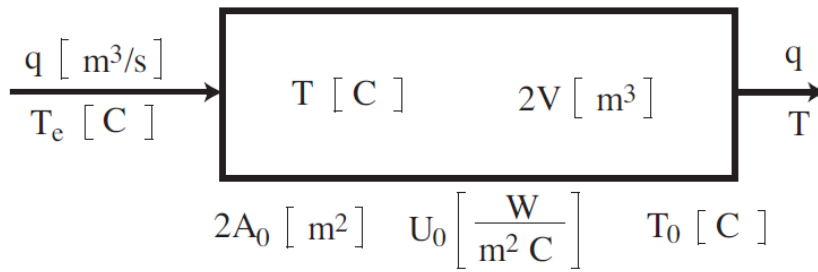
Consider the following system composed of 2 thermally coupled sections.



The feed liquid has a flow rate q and temperature T_e . The two sections are homogeneous, with a volume V and temperatures T_1 , and T_2 . There are heat exchanges between the two sections, and with the exterior. The temperatures, heat exchange surfaces and the heat transfer coefficients are indicated in the schematic. The specific heat of the liquid is $c_p [\frac{J}{kg K}]$.

- Write down the dynamic model for the system.
- Is the system linear? What if we assume that q is constant?

c) Is the system above different from the following system?



d) What type of model would we obtain if the two sections were not homogeneous?

Exercise 3

Consider the system described by the following state equations:

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 - (u_1 + u_2) & x_1(0) &= 1 \\ \dot{x}_2 &= x_1^2 - (x_2 - 1)^2 + x_1 x_2 - u_1^2 - u_2 & x_2(0) &= 1 \\ y_1 &= x_1(1 + x_2) + u_1 \\ y_2 &= x_1 + x_2 - u_2 \end{aligned}$$

Linearize this model for the equilibrium point corresponding to $\bar{u}_1 = \bar{u}_2 = 1$, for positive values of $\bar{x}_1$ and $\bar{x}_2$.